DEPARTMENT OF AGRICULTURE, CEYLON.

BULLETIN No. 66.

EXPERIMENTAL ERRORS OF FIELD TRIALS WITH HEVEA.

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EXPERIMENTAL ERRORS OF FIELD TRIALS WITH HEVEA.

CONSIDERABLE sums of money are expended annually on agricultural experiments and field trials in order to obtain reliable information of value to the practical agriculturist. The value of any experiment depends upon the confidence which may be placed on the conclusions drawn from that experiment; that is, its value depends upon the probability that similar results will be obtained if the experiment is repeated. Plot experiments and field trials are affected by so many uncontrollable factors that absolute accuracy cannot be expected. By the exercise of every possible precaution in planning and carrying out the experiment, errors may be reduced, but they cannot be entirely eradicated by these means. Some measurement of the inevitable error is therefore required, so that its probable amount may be determined, thereby ensuring that any conclusions drawn from the experiment are not vitiated by it.

One of the chief sources of error in field experiments is the natural variability of the crop. Plants are living organisms with inherent tendencies to vary, even though their environmental conditions are identical. Plants growing side by side in uniform soil under similar conditions rarely give identical yields. The effect of this factor of inherent variability can be reduced by including a large number of

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plants in a plot, but it cannot be entirely eliminated. In cropping experiments with wheat or mangolds a large number of plants are grown on a small area. For instance, an area, one-tenth of an acre, planted with mangolds 27 by 12 inches apart would contain about 1,700 plants, and a similar area of wheat would contain a greater number of plants. On a small area such as one-tenth acre the number of plants then becomes so large that inherent variability ceases to be a factor of great importance.

When, however, field trials are carried out with trees like Hevea, the use of a large number of trees to reduce the error due to variability necessitates the use of large areas. This introduces another source of error, viz., the variability of soil. Since Hevea roots penetrate deeply, it is possible that the character of the subsoil as well as that of the surface soil may affect the yields. Thus, for Hevea trials uniformity of subsoil to considerable depths as well as uniformity of soil is required. The fact that uniform surface soil may overlie very heterogeneous subsoils renders the selection of large uniform areas suitable for Hevea field trials a very difficult matter. Owing to the difficulty in obtaining large uniform areas, it becomes desirable that the experimental plots should be as small as possible consistent with accuracy, or that some method be devised by which the error due to the variability of the soil can be eliminated or measured.

These two errors, viz., the variability (i) of individual trees as regards their yield capacity, and (ii) of the soil conditions, constitute the two principal inevitable errors to which Hevea trials are subject. It is the object of the present paper to show how these errors may be measured and reduced to a minimum.

Variability of Hevea Yields.

It is well known that individual Hevea trees growing under similar conditions vary considerably in the amount of dry rubber produced. A good yielder may be growing alongside a poor yielder in the same soil. The difference in yield capacity must, therefore, be due to some inherent character of the tree, and not solely to environmental conditions. Published records of the yields of individual trees all show that great differences in yield may occur between trees growing under similar conditions.

Table 1.

Gms.	Gms.	Gms.	Gms.	Gms.	Gms.	Gms.	Gms.
1136	1478	1623	1703	1843	1987	2113	2271
1278	1482	1624	1704	1848	1991	2125	2276
1289	1485	1625	1711	1849	2014	2130	2300
1309	1487	1626	1715	1882	2022	2133	2328
1310	1497	1635	1720	1886	2028	2139	2352
1317	1505	1636	1742	1895	2029	2147	2352
1326	1507	1644	1761	1903	2034	2156	2366
1348	1522	1650	1770	1911	2048	2162	2380
1353	1523	1650	1775	1916	2053	2167	2403
1369	1531	1656	1788	1921	2056	2181	2404
1370	1536	1659	1793	1928	2063	2189	2436
1397	1541	1659	1799	1928	2079	2211	2442
1398	1546	1664	1803	1932	2079	2218	2475
1421	1548	1679	1805	1937	2084	2226	2496
1422	1548	1681	1805	1968	2096	2228	2538
1423	1558	1682	1816	1974	2099	2240	2555
1435	1569	1683	1822	1980	2099	2246	2614
1437	1579	1694	1828	1982	2104	2256	2653
1457	1581	1695	1833	1982	2110	2269	2699
1459	1585	1701	1840	1983	2111	2270	3044
	1611						

The individual yields of 161 nine-year old Hevea trees grown at Peradeniya on apparently uniform soil were recorded for the period April, 1921, to January, 1922. Full details concerning the plot were given in a previous bulletin (5).* The total yield from the whole plot was 301066 gms., and the average yield per tree 1870 gms. The yield of dry rubber of each tree for the period is given in Table 1. For convenience, these yields are arranged in ascending order, so the position in the table is in no way associated with the position of the tree in the plot.

From this table it will be seen that the yield per tree varies from 1136 gms. to 3044 gms., and that there are approximately as many trees with yields greater than the mean as there are with yields less than the mean. It will also be evident that many trees have yields approximating the average, and that as the divergence from the average becomes greater, the number of trees become smaller. This latter point may be better seen in Fig. 1, in which the yields have been plotted in the form of a frequency polygon. In plotting

* Reference is made by number to "Literature Cited," p. 22.

this polygon the yields were first grouped into convenient classes, viz., 200 gms., and the number of trees in each class, *i.e.*, the class frequency, determined. These data are given below in Table 2.

Table 2.

Yield in Grammes.		Class. Grammes.		Frequency.
1000–1199	..	1100	..	1
1200–1399	..	1300	..	12
1400–1599	..	1500	..	27
1600–1799	..	1700	..	33
1800–1999	..	1900	..	30
2000–2199	..	2100	..	29
2200–2399	..	2300	..	17
2400–2599	..	2500	..	8
2600–2799	..	2700	..	3
2800–2999	..	2900	..	0
3000–3199	..	3100	..	1

Thus, there are 12 trees having yields of over 1200 gms. and under 1400 gms.; these are placed in the 1300 gms. class, 1300 being the midpoint of that class. Similarly, there are 27 trees in the 1500 gms. class, *i.e.*, which have yields of over 1400 gms., but under 1600 gms. Along the base line of Fig. 1 intervals were marked off to represent the different classes, and verticals erected to represent the number of trees in each class, the length of the vertical being proportionate to the number of trees. The tops of the verticals were then joined by straight lines. From the resulting polygon so formed it will be readily seen that the greater number of trees have yields near the average (a vertical, M, has been erected to indicate the position of the mean or average, 1870 gms.); and that the numbers fall away fairly symmetrically on each side as the divergence from the average increases.

In the same figure is given the theoretical "normal" curve calculated to fit the present observations. Such a curve would be obtained if the material were truly homogeneous, if there were no systematic error due to bias in sampling, and if the number of trees used were sufficiently large to allow the laws of chance to operate normally. It will be seen that the frequency polygon constructed from the actual observations agrees very closely with the theoretical normal curve, considering the small number of trees used.

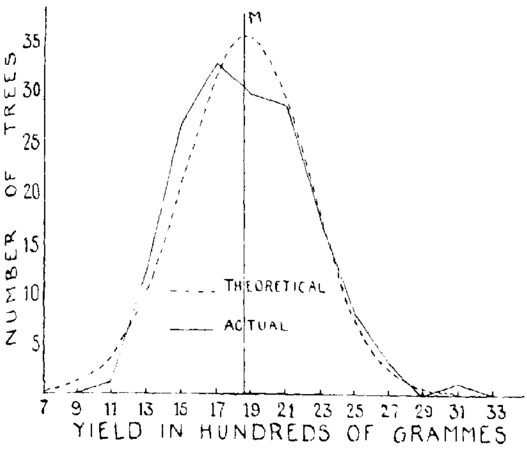

Fig. 1. – Frequency polygon and theoretical normal curve for yield of 161 Hevea trees.

PROBABLE ERROR.

The average yield of a rubber tree under the conditions of this experiment is 1870 gms., and it has been noted that the greater number of individual trees from which this mean has been obtained have yields approximating this value. The importance which may be attached to any mean value depends upon the range of variation and the extent to which the individual observations cluster round the observed mean. The more the individual observations cluster round the mean and the smaller the range of variation, the more important becomes the mean value. The extent to which the individual observations cluster round the mean may readily be seen in a graph such as Fig. 1; the more they cluster, the steeper becomes the curve; but if the range of variation is great and the dispersal even, the curve becomes flatter. Some measure of dispersion is, therefore, required which will conveniently express the amount by which any single observation taken at random is likely to deviate from the mean value.

In Fig. 2 the frequency polygon of Fig. 1 has been repeated, but the horizontal base line has been divided to represent 100 gms. classes. The number of trees in each class has been represented by dots arranged vertically above the class mark. The mean is again represented by a vertical, M, and two other verticals, E' and E'', have been erected equidistant from M, such that approximately as many dots lie between these two boundaries as lie outside them. To satisfy these conditions, E' and E'' were erected at 1630 and 2110 gms. respectively, *i.e.*, 240 gms. from the mean, so enclosing 79 dots, while 82 dots lie outside those limits. Thus, approximately half the trees have yields between 1630 and 2110 gms., and consequently there is an even chance that any individual tree selected at random will have a yield falling between these limits. Since 1630 and 2110 gms. are each 240 gms. from the mean value (1870 gms.), these limits, together with the mean value, may be readily expressed as 1870 ± 240 gms. By stating the mean yield to be 1870 ± 240 gms., it is not only indicated that the average yield per tree is 1870 gms., but also that half the values from which the average has been calculated lie within 240 gms. of that average.

The term $\pm$ 240 gms. is known as the probable error of a single result. It will be evident that the smaller the probable error, the more confidence may be placed in that result and *vice versâ*.

The probable error has been obtained above empirically. It may, however, be readily calculated from the following formula :—

$$\text{P. E. single result} = 0 \cdot 67 \sqrt{\frac{\Sigma\, d^2}{n}} \ldots\ldots\ldots\ldots (\text{I})$$

where Σd^2 represents the sum of the squares of the differences of each result from the mean and n is the number of results. Applying this formula to the present case, the probable error of a single result may be calculated to be 241·2 gms., agreeing very closely with the result obtained empirically.

As the chances are even, that any single result taken at random will not differ from the average by an amount greater than the probable error, the chances are also even that it will. It is evident, therefore, that the word "probable" in the term "probable error" is hardly used in its usual sense in this connection. The error is not the one most probable to occur, but is merely a quantity such that we may expect greater or less errors of sampling with about equal frequency provided that the frequency distribution is normal.

Table 3.

Difference from Mean in terms of Probable Error.	Odds against such Difference being due to Normal Variation.
1·00	1 to 1
1·25	3 to 2
1·44	2 to 1
1·71	3 to 1
1·90	4 to 1
2·00	9 to 2
2·05	5 to 1
2·50	10 to 1
2·93	20 to 1
3·00	22 to 1
3·20	30 to 1
4·00	140 to 1
4·90	1000 to 1
5·00	1350 to 1

If, however, two more vertical lines, 2E' and 2E'', are drawn in Fig. 2 at distances corresponding to twice the probable error, viz., 480 gms., from the mean, by counting the dots between and outside these boundaries, it will be seen that 137

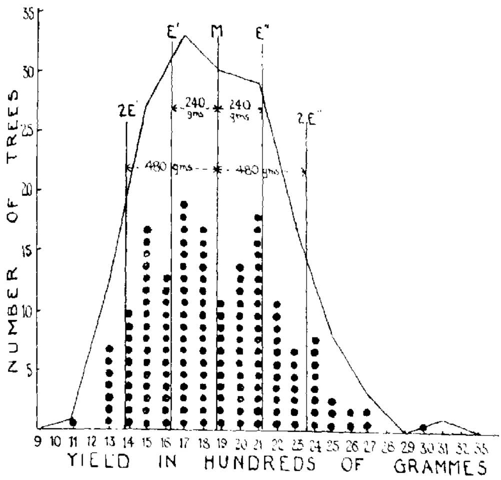

Fig. 2. -Frequency polygon for yield of 161 Hevea trees showing probable errors.

dots are enclosed, leaving only 24 dots outside these boundaries. Thus, the chances are 137 to 24 that any single result taken at random will not differ from the mean by an amount greater than twice the probable error. Or, by using the probable error as calculated from the formula, viz., 241·2 gms., the chances against a difference exceeding twice that amount may be obtained from Table 1 as follows. Twice the probable error is 482·4 gms. The number of trees with yields ranging from 1387·6 gms. (*i.e.*, 1870–482·4 gms.) to 2352·4 gms. (*i.e.* 1870 + 482·4 gms.) may be seen from Table 1 to be 136, leaving 25 outside those limits. The odds, therefore, are roughly 5 to 1 that the difference will not exceed twice the probable error. The theoretical odds (12) calculated from a normal frequency curve are given in Table 3, from which it will be seen that the theoretical odds are 4·5 to 1 against a difference exceeding twice the probable error. The same table shows that the odds are 22 to 1 against a difference exceeding 3 times the probable error. Of 161 results, therefore, only 7 should differ from the mean by more than 3 times the probable error. From Table 1 it may be seen that only 5 trees have yields which differ from the mean (1870 gms.) by more than three times the probable error (*i.e.*, 723·6 gms.). It is evident, therefore, from the above examples, that the odds calculated theoretically fairly represent what actually occurs in practice.

Applicability to Plot Experiments.

Now if, for the sake of argument, we regard the experimental area as consisting of 161 plots each containing 1 tree, then the chances are even that the yield of any one plot will differ from the normal or average yield by 241·2 gms., but the chances are 9 to 2 against it differing by more than 482·4 gms., and 22 to 1 against it differing by more than 723·6 gms. Supposing that one plot had been used to test the effect of a certain treatment, and that plot had given a yield 240 gms. better or worse than the average, it is obvious that we could not conclude that the difference in yield was due to the treatment applied, for we have seen that it is probable that any single plot would show a difference in yield of that magnitude 5 times out of 10, even when it had received no differential treatment. Similarly, a difference in yield of 720 gms. from the normal might be obtained once out of 23 times without the plot having received differential treatment.

It is still a common practice, when conducting agricultural experiments to determine the beneficial value of manurial or other treatment, to divide the experimental area into a number of equal-sized plots, one of which is kept as a control, *i.e.*, receives no treatment, while of the others, one is used to test the effect of each treatment experimented with. Any differences in yield in the treated plots from the control plot are considered to be due directly to the treatment the plots have undergone. This presupposes that the control plot is an average plot, and that all the plots, had they been treated alike, would have given approximately identical yields. It has already been shown that if the plots consisted of one tree only, the chances are even that any plot selected at random as a control plot would have a yield differing from the average by more than 241·2 gms.; and also where the plots are treated alike, they give by no means identical yields. What is true of a single tree plot is also true to a great extent of larger plots. This has been shown by many workers with various agricultural products, but one experiment only need be mentioned here to show this point.

Bishop, Grantham, and Knapp (3) selected ten plots of Hevea, each of 5 acres. "Great care was taken to find an area where the trees were to all appearances the most uniform. The land was practically level, and no great differences were apparent in soil and drainage." The trees were all tapped and in every way treated alike, and care was taken to eliminate any error due to the tapping cooly. Over a period of ten months the worst plot gave a yield of 838·1 lb. of dry rubber, while the best gave 953·2 lb., the average yield being 898·1 lb. Thus, the worst plot gave a yield 60 lb., or 6·7 per cent., less than the average, while the best plot gave a yield 55 lb., or 6·1 per cent. better than the average. Had these plots been used to determine the effect of differential treatment and one by chance had become the control plot, then the difference in yields of 115 lb., or over 12 per cent., would have been attributed to the treatment, whereas under the conditions of the experiment it occurred without differential treatment.

It is evident, therefore, that experiments carried out on large plots, such as 5-acre plots, are liable to errors from the same causes as occur with small plots. Since the use of large plots does not eliminate the errors due to natural variation, it becomes necessary to determine whether the size of the error is reduced as the size of the plot is increased, and if so, with what sized plot is the error reduced to a minimum.

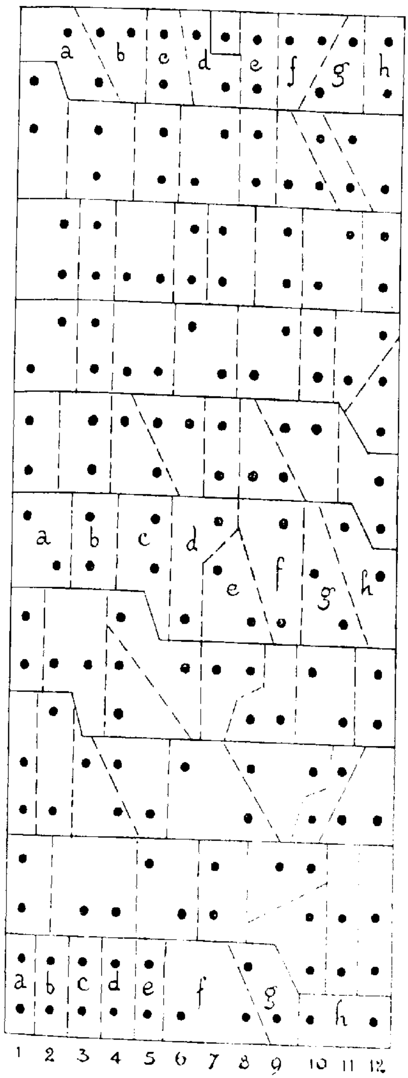

Fig. 3.—Plan of experimental plot showing the division of the area into plots of various sizes.

Size of Plot.

If we regard each of the 161 trees as being a separate plot, then it is possible by combining adjacent plots to obtain plots of 2, 4, 8, 16, and 32 trees. In Fig. 3 is given a plan of the experimental plot, showing how adjacent single trees were combined to form 2-tree plots. Each 2-tree plot is marked *a* to *h*. By combining plots *a* with the adjacent plots *b*, *c* with *d*, *e* with *f*, and *g* with *h*, 4-tree plots are obtained. By combining the plots *ab* with the adjacent plots *cd*, 8-tree plots are obtained; similarly, *efgh* form 8-tree plots. By combining these 8-tree plots together, 16-tree plots, each of which extends across the experimental area, are formed. Two adjacent 16-tree plots form a 32-tree plot. When combining plots in this manner, it was evident that one tree had to be omitted from the plots, as 160 trees was a more convenient number to work with than 161. This tree was selected at random, No. 91 being selected and consequently omitted when forming 2-tree plots.

Table 4.

Size of Plot.	Extreme Yields per Plot.	Mean Yield per Plot.	Probable Error.	Probable Error as a Percentage of the Mean.	Theoretical Probable Error.
	Gms.	Gms.	Gms.		
1 tree ..	1136–3044	.. 1870 ..	241 ..	12·9 ..	12·9
2 trees ..	2598–4907	.. 3743 ..	356 ..	9·5 ..	9·1
4 trees ..	5419–8890	.. 7487 ..	525 ..	7·0 ..	6·4
8 trees ..	12853–17298	.. 14973 ..	674 ..	4·5 ..	4·6
16 trees ..	26723–31437	.. 29947 ..	981 ..	3·3 ..	3·2
32 trees ..	56300–61680	.. 59893 ..	1343 ..	2·3 ..	2·3

The mean yield and probable error (using formula (I)) were then calculated for the 2-tree plots. Similar determinations were made for the 4-, 8-, 16-, and 32-tree plots, and the results are given in Table 4. The probable errors are given in grammes, and also expressed as percentages of the mean, in which form they can be more readily compared. It will be seen from the table that, as the size of the plot is increased, the probable error attaching to the result obtained for a single plot diminishes.

The experimental plot was divided again into 2-, 4-, 8-, 16-, and 32-tree plots, but this time the plots were arranged so that the 32-tree plots formed narrow plots extending the length of the area, and not across it as before. The mean yield and probable errors of the plots arranged in this manner are given in Table 5. It will be noticed that the results obtained from this arrangement of plots agree very closely with those obtained from the first arrangement.

Table 5.

Size of Plot.	Extreme Yields per Plot.	Mean Yield per Plot.	Probable Error.	Probable Error as a Percentage of the Mean.
	Gms.	Gms.	Gms.	
1 tree ..	1136–3044 ..	1870 ..	241 ..	12·9
2 trees ..	2598–4970 ..	3743 ..	352 ..	9·4
4 trees ..	6125–9381 ..	7487 ..	477 ..	6·4
8 trees ..	13256–17809 ..	14973 ..	749 ..	5·0
16 trees ..	27159–32907 ..	29947 ..	1039 ..	3·5
32 trees ..	56661–62938 ..	59893 ..	1614 ..	2·7

If we plot the probable error expressed as a percentage of the mean from Table 4 against the size of the plot, as is done in Fig. 4, it may be seen that, as the plot increases in size, the reduction of the probable error is at first rapid, but that later the reduction is small. In the same figure is also drawn a theoretical curve based purely on statistical grounds to show the percentage probable error of successively larger plots consisting of a number of trees taken at random, *i.e.*, not necessarily adjacent nor forming a regular plot. The curve for the experimental results agrees very closely with the theoretical curve. It will be noticed that at the 16-tree plot the curve has begun to flatten out, and that beyond this point it begins to approximate the horizontal. From this we may conclude that there is a rapid reduction in the probable error as progress is made from a 1-tree to a 16-tree plot. By increasing the size of the plot above 16 adjacent trees the reduction of the probable error, in comparison with the number of extra trees required, is small. It would, therefore, appear not profitable to increase the size of the plot beyond 16 trees, as any increase in size of the plot beyond this amount is not accompanied by any appreciable reduction in the probable error.

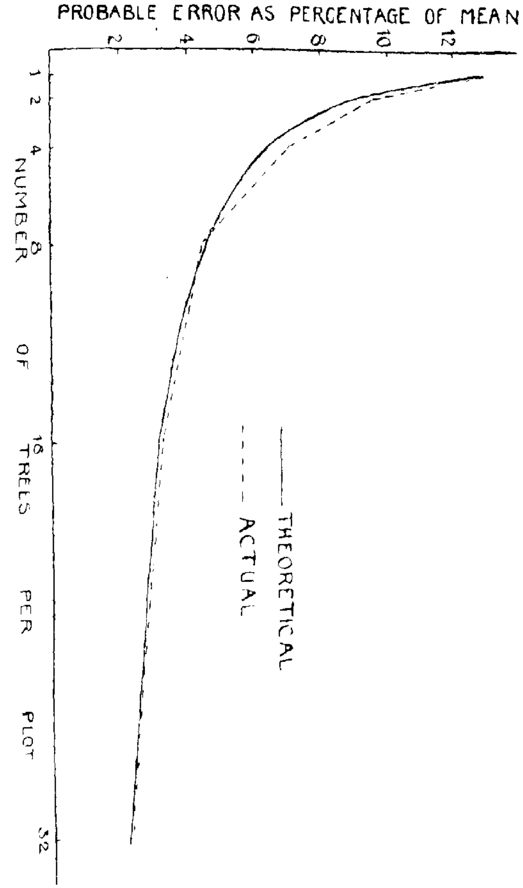

Fig. 4.—Graph showing the effect, actual and theoretical, of increasing the number of trees per plot on the size of the probable error.

Use of the Probable Error in interpreting Results.

Assuming that the experimental area had been laid out in 16-tree plots as in Fig. 3, that one plot had been kept as a control, and the others had received differential treatment, what differences in yield may be expected to occur because of the varying yields of the trees, and what differences may safely be attributed to the differential treatment ?

The probable error of a 16-tree plot was found to be $\pm$ 3·3 per cent. for this arrangement of the plots (Table 4). That is, the chances are even that the yield of the control plot will fall within $\pm$ 3·3 per cent. above or below the true mean of plots of that size. Similarly, any other plot, had it not received differential treatment, would have given a yield of which the chances are even that it would fall within the accuracy of $\pm$ 3·3 per cent. of the mean. In comparing the yields of these two plots, each of which has a probable error of 3·3 per cent., their difference will be liable to a greater error. In fact, the probable error of the difference between two results is $\sqrt{2}$ times the probable error of one result.*

In this case the probable error of the difference in yields between two plots will be $\sqrt{2} \times 3{\cdot}3$ per cent. = 4·6 per cent. Therefore, if a plot undergoing differential treatment differs in yield from the control by only 4·6 per cent., the chances are even that that difference is not due to the differential treatment, or, in other words, half the time that difference in yield may be due to the treatment and half the time it may be due to casual variation. Consequently we are not assured beyond an even chance that the difference in yield is real and due to the differential treatment. So slight an assurance could not be expected to prompt a planter to change his method of tapping or cultural treatment. If, however, a plot gave a yield 13·8 per cent. better than the control, i.e., if the difference in yields is 3 times that of its probable error, we know from Table 3 that the chances are 22 to 1 against that difference being due to casual variation. If we conclude that this difference was due to the treatment, we may expect to be correct 22 times out of 23 and wrong once out of 23 times.

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* Where two results have to be compared, each of which has a different probable error, the probable error of their difference is obtained from the formula—

P. E. difference $= \sqrt{a^2 + b^2}$......(II).

Where a is the P. E. of one result and b the P. E. of the other.

When basing conclusions on the results of an experiment, it must be decided what odds may be accepted as amounting to certainty. In making this decision, much will depend upon the nature of the experiment. For manurial experiments odds of 30 to 1 are commonly accepted as amounting to certainty, *i.e.*, any difference in yield must be 3·2 times as great as its probable error (*vide* Table 3). For some experiments we may admit smaller odds and for others, depending upon the nature of the experiment, greater odds may be demanded. Reitz and Smith's statement (1) concerning the significance of probable errors is of interest. "If the difference between two results does not exceed 2 or 3 times the probable error, the difference may reasonably be attributed to random sampling. If the difference between two results is as much as 5–10 times the probable error, the probability of such differences in random sampling is so small that we are justified in saying that the difference is significant. In fact, a difference of 10 times its probable error is certainly significant in so far as there is certainty in human affairs."

If, therefore, experiments are carried out on 16-tree plots, of which the probable error is 3·3 per cent. and consequently of which the probable error of differences in yield between plots is 4·6 per cent., and if odds of 30 to 1 are demanded as amounting to certainty, then any difference in yield to be significant must be 3·2 times as great as its probable error, *i.e.*, 14·7 per cent. Any difference less than that amount does not give sufficient assurance that it is due to the treatment the plot has received, and not to the normal variability of yield of the plots. If the experiment had been carried out to determine whether a system of manuring which would increase the cost of production by 5 per cent. would pay, the experiment must show an improved yield of 14·7 per cent. before it could safely be concluded that the manuring had *any* effect, and 20 per cent. to justify the conclusion that the system of manuring would pay. This is a large difference to expect from any experiment, and in many cases it may be necessary to reduce the probable error so that reliable conclusions can be based on much smaller differences.

Duplication of Plots.

It will be remembered that by increasing the number of trees in a plot the probable error was decreased, but not in direct proportion to the increase in the number of trees. The probable error of a 1-tree plot was found to be 12·9 per cent., and that of a 4-tree plot 6·4 per cent. (Table 4). Though the plot has been increased in size 4 times, the probable

error has been reduced only by half. Similarly, a 2-tree plot has a probable error of 9·4 per cent., whereas a plot 16 times as large, *i.e.*, a 32-tree plot, has its probable error reduced to about one-quarter, viz., 2·7 per cent. (Table 5). If, therefore, it is required to increase the reliability of the experiment 4 times, *i.e.*, to reduce the probable error to one-quarter, it is necessary to use, not 4 times, but, 16 times as many trees. In other words, the reliability of a determination increases, not in proportion to the number of trees, but in proportion to the square root of the numbers.*

If, therefore, it is required that the probable error of the experiment shall not exceed 1·1 per cent. (*i.e.*, one-third of the error given by a 16-tree plot), so that a difference in yield of 5 per cent. will give a significant result, 9 times as many trees, *i.e.*, 144 trees per plot will be necessary.

Theoretically it is immaterial whether the 144 trees are situated together in one plot or are scattered in smaller units, say, 9 plots each of 16 trees, over the experimental area. In practice, however, it is of great importance. Theoretically we assume that the soil conditions over the whole area are uniform, and that we are dealing with one variable only, viz., the variability in yield of the trees. Two plots, each of 144 trees, would occupy nearly 3 acres of ground, and in practice it would be found well-nigh impossible to obtain 3 acres of absolutely uniform soil suitable for the experiment. Soils which even to an expert appear uniform are found on trial not to be uniform. The lack of uniformity of the soil introduces a new error, but it will be shown that by distributing small equal-sized plots over the experimental area, and using alternate plots for the control, the soil error may to a great extent be eliminated. Thus, it is better to use 9 scattered 16-tree plots than one plot of 144 adjacent trees, for in the former case it is more likely that plots will occur on a fair proportion of both high- and low-yielding areas than in the latter case.

Variation of Soils.

The literature of variety tests and field experiments abounds in illustrations of the fact that it is practically impossible to secure absolutely uniform tracts of land for field trials. The lack of uniformity of the soil, or soil heterogeneity, results in a lack of uniformity of yields of the plants

* This is expressed by the formula—

$$\text{P. E. average of } n \text{ results} = \frac{\text{P. E. single result}}{\sqrt{n}} \qquad \text{(III)}.$$

growing on that area. Those growing on the more fertile parts may be expected to give greater yields than those growing on the less fertile parts. Soil heterogeneity will consequently influence the final results, and this influence may of itself be greater than that exerted by the one controlled variable for which the experiment was undertaken. Some account must, therefore, be taken of this factor of soil heterogeneity, and some precautions to reduce the influence of this factor to a minimum, and to ensure that its influence upon the numerical results of the experiment will be less than that of the factors in crop production which the investigation is seeking to compare.

Harris (7) has proposed a method of measuring the heterogeneity of the soil of a field, and has used his method to analyse the actual yields of test plots selected by agricultural experts as being suitable for experiments. The plots were selected with great care and thoroughness as being satisfactorily uniform. Yet a mathematical analysis of the resulting crop from the plots showed that in every field the irregularities of the substratum were sufficient to influence, and often profoundly, the experimental results. This merely illustrates the inadequacy of personal judgment concerning the uniformity in crop-producing capacity of fields under consideration for experimental work.

Further, by an analysis of the results of determinations of the water content at different levels for the upper 6 feet of an experimental field, Harris showed that heterogeneity in this respect is least at the surface, and greatest at a depth of 4 feet. The surface layer of soil might, therefore, be apparently uniform as regards water content, while the underlying layers might differ greatly from one part of the field to another. In other words, an apparently uniform soil may overlie a heterogeneous subsoil.

Similarly, an analysis of the determinations of nitrogen and carbon in soil from adjacent borings showed that heterogeneity occurred as regards these characters. There can, therefore, be little doubt that the heterogeneity of experimental fields in their capacity for crop production is due to a large extent to the lack of uniformity of the physical and chemical characters of the soil.

Examination of the published results of plot experiments shows that, as a rule, the fertility of the soil changes gradually from one side of a field to the other. This, however, is not always the case, for in some fields the soil is obviously patchy, fertile and poor soils occurring in areas irregularly

dotted about the field. Usually the change from a good to a poor soil is gradual, the good soil grading off through medium to poor soils.

In view of the work done by Harris and other workers on soil heterogeneity, which has been summarized above, it is not surprising to find that the soil of the small experimental area under consideration, though to all appearances very uniform, exhibits heterogeneity when the crop yields are analysed in the manner devised by Harris. The heterogeneity of the soil as exhibited by the variation in yield of the trees is shown diagrammatically in Fig. 5. The plot consisted of 12 rows of trees running approximately east to west. The rows, for convenience, are numbered 1–12, the row on the southern side being No. 1, and that on the northern side No. 12. The mean yield per tree for each row was determined and the results shown graphically in Fig. 5. It will be seen that in passing from row 1 to row 6 there is a general gradual increase in the average yield per tree, indicating a general improvement of the soil and other environmental conditions. From row 6 to row 10 there is a general gradual decrease in yield ; this is followed by an increase to row 12. This roughly indicates that soil conditions for rows 6 and 12 are rather better than those for rows 2 and 10, and that in passing across the plot from north to south there is a slight variation in the soil.

The theoretical curve of Fig. 4, representing the probable error for plots of various sizes, is founded on the assumption that no variation in soil conditions occurs on the plot. The fact that the actual probable errors for the 16-tree and 32-tree plots arranged across the area (Table 4) agrees closely with the theoretical results indicates that those plots are uniform, *i.e.*, each contains a fair proportion of good and bad soils, so that the effect of soil heterogeneity is eliminated. Plots of similar size arranged lengthwise in the field (Table 5) show greater probable errors, indicating that in these plots there is a slight variation of soil conditions. From this we might conclude that the soil varies in a direction north to south across the field, as by arranging plots parallel in this direction relatively uniform plots are obtained. This agrees with the conclusion arrived at from consideration of Fig. 5.

The fact that the probable errors for actual plots on this area do not diverge to any great extent from the theoretical results indicates that, though soil heterogeneity occurs, it is not very pronounced. This is to a large extent due to the small size of the experimental area. Where similar experiments have been conducted on larger areas which, despite the care with which the sites have been chosen, are consequently

more heterogeneous as regards soil conditions, the divergence of the observed probable error of large plots from the theoretical is more pronounced.

Grantham and Knapp (6) used an area of 12½ acres of 5-year old rubber, planted 120 trees to the acre and tapped daily on one-third circumference, for their experiments. All the trees were treated alike, and precautions were taken to eliminate errors due to tapping coolies, &c. Individual tree records were kept, and calculations made as to the probable errors occurring when the area was divided into plots of various sizes and shapes. They found that the size of the probable error for plots of definite size varied with the shape of the plot. Where the long axis of the plot ran north to south, the probable error was greater than that of plots of the same size where the long axis ran east to west; and plots which were square or nearly so gave intermediate probable errors. This would indicate a distinct heterogeneity of the soil, the soil conditions changing in a direction paralled to the plots with long axis east and west.

Table 6.

	Actual Probable Error. Per Cent.	Theoretical Probable Error. Per Cent.
Single trees	40·4	42·7
5-tree	19·1	19·1
25-tree square	9·5	8·6
50-tree rectangular 10 by 5	6·9	6·1
100-tree square	5·4	4·3
200-tree rectangular 8 by 25	4·5	3·0
100-trees 4 scattered lines of 25 trees east to west	4·3	4·3
200-trees 8 scattered lines of 25 trees east to west	3·6	3·0

In Table 6 are repeated a few of their results. It will be noted that the actual probable errors diverge from the theoretical, and that as the size of the plot increases the divergence becomes greater. This divergence is attributed by the authors to "the introduction of other variants, such as site variation," which is the same as is implied in the present paper by soil variation. When, however, four scattered plots, each of 25 trees, are used instead of a single plot of 100 trees, the probable error is reduced from 5·4 per cent. to the theoretical one of 4·3 per cent. When 200 trees are used, divided up into eight scattered plots each of 25 trees, the probable error becomes 3·6 per cent. instead of 4·5 per cent. for a single plot of 200 trees. Evidently, therefore, a smaller probable error is obtained by duplicating small plots than by using the same number of trees grouped into one plot.

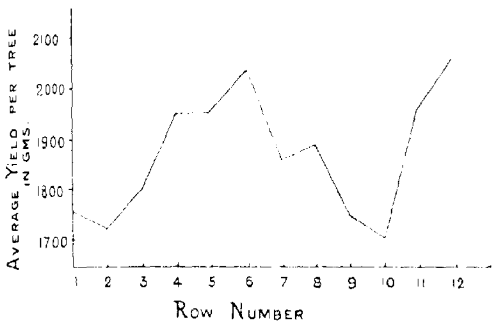

Fig. 5.—Graph showing the variation of the mean yield per tree for each row of the experimental area.

The same fact has been shown by numerous experimenters working with other crops. Batchelor and Reed (2) working with fruit trees (oranges, lemons, walnuts, and apples) have shown that the probable error obtained by using six scattered plots each of 4 trees is smaller than that obtained from three scattered plots each of 8 trees, which in turn is smaller than that obtained from a single plot of 24 adjacent trees. In Fig. 6 some of their results are represented graphically. It will be seen that there is a reduction in the size of the probable error as the number of trees in a plot is increased (*cf.* Fig. 4). The actual curve for adjacent trees, however, does not closely fit the theoretical owing to variation in soil conditions over the experimental area. Where, however, the plots are made up of 8-tree units, there is a greater reduction of the probable error than occurs when the plots consist of adjacent trees; and where they consist of 4-tree units, the reduction is still greater, and the probable errors closely approximate the theoretical. A large number of unit plots thus gives a more typical sample of the area than half as many units with twice the number of trees in a unit. As a general rule, therefore, the duplication of plots is always advisable, for this is the surest way of eliminating errors due to soil heterogeneity.

Optimum Size of Plot.

If, then, the size of the plots is kept as small as possible, it will be evident that the actual size of the unit plot must depend to a large extent upon the nature of the experiment itself. For instance, if it is required to determine the relative merits of two systems of tapping, it would be practicable to tap alternate trees by one method, and the intervening trees by the other. In this way the effect of any variation in the fertility of the soil would be eliminated, and we should be sure that the probable error of the experiment was reduced to a minimum. If, however, the experiment was to test cultural methods or the merits of fertilizers, the use of single tree plots would be impracticable. For such trials a plot containing about 8 trees would probably be the smallest practical unit.

Uniformity of Probable Errors.

We have seen that the amount of confidence which may be placed in the results of any experiment depends primarily upon the probable error of that experiment. It has been ascertained that, on the Peradeniya experimental plot, the probable error of a single plot of 16 trees is 3·3 per cent. The question arises as to whether this value is constant for

all Hevea trials wherever situated. Some light can be thrown on this question by making a comparison of the results published by other workers on variation in Hevea yields.

Table 7.

Authority.	Number of Trees.	Average Yield per Tree	Probable Error Single Result.		Probable Error 100-tree Plots.
		Gms.	Gms.	Per Cent.	Per Cent.
Bobilioff (4)	491	15·6	3·9	25	2·5
Whitby (11)	1011	7·12	3·7	51	5·1
Heusser (8)	100	11·26	1·9	17	1·7
La Rue (9)	1063	1410	565·6	40	4·0
Grantham and Knapp (6)	1361	—	—	40	4·0
Peradeniya plot	161	1870	241	13	1·3

In Table 7 are given some records of the variation in yield as reported by other workers. This table shows the number of trees used by each investigator in determining the mean yield. In most cases the value of the probable error was not given, the standard deviation being used instead. Where this was given, the probable error was calculated for the purposes of this table from the formula—

P. E. single result = $\cdot 6745 \times$ standard deviation .... (IV).

The probable error is also expressed as a percentage of the mean, and the theoretical probable error for a plot of 100 trees is also given. It must be remembered, however, that the theoretical probable error of 100-tree plot is much smaller than would be actually obtained in most cases by single plots of that size, but would be approximated if the 100 trees are scattered over the experimental area as small plots. It will be seen that there is no uniformity regarding the size of the probable error. The smallest errors were obtained from the Peradeniya plot and by Heusser. In both these cases small numbers of trees were used for the experiment, but what would appear to be more important is the fact that the trees used in both cases were the offspring of selected mother trees. The trees used by La Rue were the same as those used by Grantham and Knapp, and so an agreement regarding the size of the probable error would be expected. The results given by Bobilioff, Whitby, and Heusser were obtained from tapping tests over short periods, and for that reason may not be strictly comparable with those of Grantham and Knapp and of the Peradeniya experiment.

Until further records of tapping tests over long periods are available for comparison, we cannot safely conclude that the probable error of a plot of a certain size will have a definite

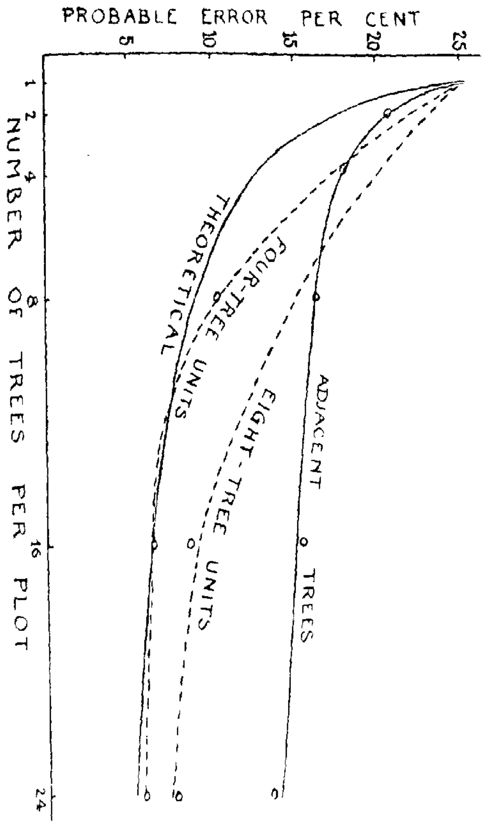

Fig. 6.--Graphs showing the reduction of the probable error by increasing the number of trees in the plot and by using small units.

value. The value of the probable error will possibly vary with the parentage of the trees, with their age, and the conditions under which they are grown.

This means that before an experiment is started, information must be obtained regarding the size of the probable errors of the experimental plots. This is best done by keeping yield records of the individual trees of the experimental area, all trees being treated and tapped alike. The information obtained from the preliminary tapping would be of great value, as from these results, not only would the size of the probable error be obtained, but also much could be ascertained as to the variation in soil fertility, the optimum size of a plot, and the best arrangement of the plots to eliminate errors due to the heterogeneity of the soil.

Where the keeping of individual tree records is impracticable, the area should be divided up into as small plots as is practicable and the yields of each individual plot recorded. The probable error attached to plots of this size may then be ascertained and the number of such plots necessary to give the required precision for each trial calculated.

Length of Tapping Experiments.

The length of time over which this preliminary tapping experiment should extend is about five months. This period has been determined from the following observations. During the first month's tapping of the Peradeniya plot the probable error of any one individual tree result was $\pm$ 26 per cent., the probable error of the yield of any one tree after three months' tapping was $\pm$ 18 per cent., and after five months' tapping $\pm$ 13 per cent. It will be noticed, therefore, that by extending the period of a tapping experiment up to five months a reduction in the probable error is obtained. An extension of time beyond five months has no effect on the probable error, for it has already been seen that after ten months the probable error still remained at 13 per cent.

Grantham and Knapp (6) found a similar reduction of the probable error with an increase of the time of tapping. They state: " There is a reduction of probable error by increasing the period from three to six months, but beyond that period no reduction." Maas (10) obtained a similar result from his analysis. He showed that the probable error of a plot diminishes as the periods of observation are increased up to five or six months, and recommends three to six months as sufficient for practical purposes. The result obtained from the Peradeniya plot, therefore, agrees very closely with those of other investigators, and it may be taken as established that five or six

months is a sufficiently long tapping period for any preliminary experiment to determine the probable error of the plots.

Since the probable error of plots is as small after five months' tapping as it is after ten or twelve months' tapping, it follows that for many experiments no greater accuracy is attained by carrying on tapping tests for periods longer than five or six months. It will be obvious that five months is not sufficiently long for all experiments, for example, for manurial tests with slow-acting fertilizers. The period to which trials should be extended must vary with the nature of the experiments, but five months should be regarded as the minimum period.

SUMMARY.

Two of the most important errors to which field trials with trees like Hevea are liable are due (i) to the variability of yields of individual trees, and (ii) to the variability of soil fertility. The errors due to these causes may be so great as to vitiate any conclusions drawn from the experiments.

The errors due to the variability of yields of individual trees could be reduced to an insignificant quantity by using large plots containing many trees, if truly homogeneous soil over the experimental area could be obtained.

Unfortunately, absolute uniformity of soil is rare. And, moreover, an apparent uniform soil may overlie heterogeneous subsoils. With deep-rooted trees like Hevea, uniformity of the subsoil is quite as important as uniformity of the surface soil.

By merely increasing the size of the plot the advantage in reducing the error due to variability of individual yield is destroyed by the introduction of the second error due to the variability of the soil. Consequently an experiment carried out on a large area with many trees is not necessarily more accurate than a similar experiment on a smaller area with fewer trees.

The error due to lack of uniformity of soil conditions is best eliminated by the use of small duplicate plots. Thus, ten plots, each of 10 trees, scattered over the experimental area, will give a more reliable result than 100 trees together in one block. By using small scattered plots a fairer sample of the soil for the whole area is obtained.

The actual number of trees required for each trial will depend mainly upon the degree of accuracy required, the greater the accuracy demanded, the larger the number of trees required. The degree of accuracy, however, varies, not in proportion to the number of trees, but in proportion to the square root of their number.

The reliability of any result is measured by means of its probable error. An attempt has been made in the foregoing pages to give a simple explanation of the meaning and use of probable errors.

At present it is not possible to say that a plot of a certain size will consequently have a probable error of a definite size. The probable error attached to a plot will depend, not only upon the size of the plot, but also on the ancestry of the trees, on their age, and possibly on their method of tapping and treatment, as well as on climatic and environmental conditions. Until further information is obtained concerning the variability of Hevea trees under various conditions, it will be necessary to determine the magnitude of the probable errors for each experimental area.

This is best done by keeping yield records of each individual tree of the area for a period of five months, each tree being treated and tapped similarly. From these records valuable information can be obtained concerning not only the size of the probable errors of plots of various sizes, but also as regards the optimum size and the best arrangement of the plots to eliminate the effect of soil heterogeneity. This information can be usefully applied to all subsequent experiments.

Where the keeping of individual tree yield records is impracticable, the area should be divided into as small plots as is practicable and the yields of each individual plot recorded. From these records the probable error attached to a plot of this size can then be calculated, and from this the number of plots necessary for each trial in order to give a certain precision can be ascertained.

Only by the use of probable errors can it be decided whether small differences in yield shown by trials are due to the effect of treatment or to inherent differences of the trees. Where the differences are great, the use of statistical methods may be unnecessary in arriving at a conclusion. But as it is not known till the end of the experiment whether the difference is small or great, it is advisable to determine before commencing the experiment the probable amount of error to which the experiment is liable. Some error is inevitable, and without some knowledge as to the size of that error, conclusions based on an experiment may be unreliable.

Much that has been written concerning experimentation with Hevea yields is equally applicable to other tropical crops, such as tea and coconuts.

C. H. GADD.

October, 1922.

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